

Table of Outcomes with Two Dice

When working with Probabilities, most of the time it's simpler to use Formulas: Probability of Success equals Probability of rolling with two Dice, at least one a "One" or a "Five": $Ps [2] = 2 \times Ps [1] - Ps [1]^2$. This equation deals with Adding individual Probabilities and subtracting the "Overlap". But what if we approach this Problem by a grunt method: Layout a Table of Outcomes and simply count the Successes:

Outcomes for Die #1	Outcomes for Die #2					
	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

We count 20 Successes out of 36 with at least one "One" or a "Five": $Ps [2] = 20/36$. If we were to simply add the Probabilities without regard for Overlap, we get the errant $2/3$. What happened is that Die # 1 "claimed" the *whole* First and Fifth Row as Successes, and Die #2 "claimed" the *whole* First and Fifth Columns as the same. By adding the Successes of each Die, we also added the Overlap *twice*, and so now we need to subtract *one* Overlap to correct the over-count:

$$\begin{aligned}
 Ps [2] &= 2 \times Ps [1] - Ps [1]^2 \\
 &= 2 \times 1/3 - (1/3 \times 1/3) \\
 &= 6/9 - 1/9 = 5/9
 \end{aligned}$$

(which corroborates since $5/9 = 20/36$)